# Topologically decoherence-protected qubits with trapped ions

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We show that trapped ions can be used to simulate a highly symmetrical Hamiltonian with eingenstates naturally protected against local sources of decoherence. This Hamiltonian involves long range coupling between particles and provides a more efficient protection than nearest neighbor models discussed in previous works. Our results open the perspective of experimentally realizing in controlled atomic systems, complex entangled states with decoherence times up to nine orders of magnitude longer than isolated quantum systems.



It is universally accepted that quantum computation would be able to solve diverse classes of hard problems more efficiently than its classical counterpart [1, 2]. However, its practical realization is made difficult by the conflicting requirements imposed by the absence of decoherence, qubit manipulation and scaling [3]. Generally, larger dimensional systems are in general more sensitive to decoherence, so that scaling of quantum information processors is a huge experimental challenge. In the present work, we show how to avoid decoherence and protect complex multiparticle quantum states (that can define a qubit) from its effects in an experimentally realizable trapped ion system.

There are two conceptually different approaches to suppress the decoherence to a level required for quantum error correction. One is to reduce directly the physical noise that leads to decoherence. It is however very difficult due to the stringent requirements imposed by quantum error correction [4]. Another is to encode qubits in decoherence free subspaces (DFS) which are decoupled (at least to first order) to the dominant sources of noise. The efficiency of the latter strategy is demonstrated by experiments showing a dramatic increase in the decoherence times in diverse physical systems: atom clouds [5], trapped ions [6] or superconducting circuits [7] in which noise mostly originates in the fluctuating electric or magnetic fields. In trapped ion systems, DFS are usually hyperfine states that can either be symmetrically coupled to the magnetic field [8], suppressing dephasing to first order, or have the same energy [9], avoiding spontaneous emission.

Here we propose a design that provides higher degree decoupling from environment. The main idea [11, 12, 13, 14, 15, 16, 17] of our approach is to implement a long range Hamiltonian which non-local symmetries ensure that not only the linear but also quadratic, cubic, etc coupling to the environment of the lowest doublet vanishes, providing thereby a high degree of protection against the effects of most sources of physical noise. The Hamiltonian introduced here is more efficient for decoherence protection than previously studied ones, as is shown below. In our scheme, the protected states are complex entangled states of $N^2$ spin 1/2 like particles. It is an example of multidimensional quantum state that is far more robust against decoherence than its individual elements.

In the second part of this letter we propose the implementation of the model Hamiltonian in trapped ion systems. The long range interaction responsible for protection in our Hamiltonian is particularly adapted to trapped ions systems. We show how this type of experimental system that has been already employed to implement basic operations of quantum logics [10] can be also used to realize the protection by topological order.

We start by introducing the long range Hamiltonian acting in a collection of two level systems.

$$H = -J_x \sum_i^N \left( \sum_j^N \sigma_{i,j}^x \right)^2 - J_y \sum_j^N \left( \sum_i^N \sigma_{i,j}^y \right)^2 . \quad (1)$$

Here we label each particle by its position in a two dimensional lattice: $\sigma_{i,j}^{x,y}$ are Pauli matrices of the spin situated at the intersection of the $i^{th}$ row and $j^{th}$ column. This Hamiltonian couples the particles in the same row by $\sigma_{i,j}^x \sigma_{i,k}^x$ interaction and those in the same column by $\sigma_{i,k}^y \sigma_{j,k}^y$ interaction. Consider the projections of all spins in a given row onto $y$-axis. Evidently the projection is preserved by the column interaction while row interaction might change it by 0 or 2 units. Thus all terms in the Hamiltonian preserve the parity of the $y-$projection of the spins in one row. Mathematically, it can be described by non-local symmetry transformations generated by the operators $P_i = \prod_j \sigma_{i,j}^y$ and $Q_j = \prod_i \sigma_{i,j}^x$ that involve product of all spins in each row or column. It can be easily shown that the operators of a given set commute with all others in the same set, *i.e.* $[P_i, P_j] = 0 = [Q_i, Q_j]$ but anti-commute with all the operators of another sets, *i.e.* $P_i Q_j + Q_j P_i = 0$. They also satisfy $P_i^2 = 1 = Q_j^2, \ \forall i, j$

and $[H, P_i(Q_j)] = 0$. These commutation relations imply the existence of doubly degenerate states, combining the advantages of previously introduced DFS [6, 8, 9]. Indeed, a state corresponding to an even value of the projection along the $y$-axis along one row is converted into an odd parity state by the action of $Q_j$ operator that commutes with the Hamiltonian. Finally, the numerical diagonalization of the Hamiltonian (1) shows that the two ground states are separated from the rest of the spectrum by a gap $\Delta$ which depends weakly on the size of the system.

We now discuss the effect of a generic local noise described by the operator

$$\mathcal{N} = \sum_{i,j}^{N^2} (b^x_{i,j}\sigma^x_{i,j} + b^y_{i,j}\sigma^y_{i,j} + b^z_{i,j}\sigma^z_{i,j}), \qquad (2)$$

where the $b^{x,y,z}_{i,j}$ are arbitrary time-dependent coefficients. Note that no other hypothesis is needed concerning the noise besides its local character: this noise can be due to energy fluctuations caused by random fields acting on the atomic system (described by $\hat{\sigma}_z$ operators) as well as spontaneous emission (described by $\hat{\sigma}_-$ operators). Equation (2) should be added to the model Hamiltonian (1). Individual terms of (2) do not commute with some of the symmetries, $P_i$ and $Q_j$, partially lifting the ground state degeneracy. When only $k < N$ such terms are present $2N - 2k$ symmetries remain intact ensuring the degeneracy of the ground states levels. Thus, decoherence is due to the simultaneous action of $N$ noise terms, in a processes taking place in the $N^{th}$ order perturbation theory. The small parameter controlling the perturbation development is $\max(b^{x,y,z}_{i,j})/\Delta$. Decoherence rate acquires an additional small factor of $\left(\max(b^{x,y,z}_{i,j})/\Delta\right)^{N-1}$ when $N$ noise terms act simultaneously. Note that the decoherence rate of a system consisting of $N^2$ interacting particles usually increases with the number of particles, a behavior which is in clear contrast to the one observed here. The resulting decoherence rate is thus

$$\Gamma_p = \alpha_N \Gamma_0 \left(\max(b^{x,y,z}_{i,j})/\Delta\right)^{N-1}, \qquad (3)$$

where $\alpha_N$ is a numerical constant of the order of the unity. For this protection to be efficient the gap $\Delta$ should remain large as $N$ increases.

The results of the exact numerical diagonalization of the long range Hamiltonian (1) with $J_x = J_y$ reported in Tab. I show that this is indeed the case. This property of (1) is a clear advantage over short range models considered in [13, 14, 15, 16]. It implies that the suppression of the noise is more efficient in the present system. Indeed, numerical calculations show that the static fields $b^{x,y,z}_{i,j}$ randomly distributed in the interval $(-0.1, 0.1)J$ result in a very small splitting of the degenerate doublet $\delta E \sim 10^{-4} - 10^{-5}J$ , two orders of magnitude better than that of [14] in agreement with the estimates above.

We now discuss possible implementations of the mathematical model (1) in a physical system. As was shown previously, short range Hamiltonians with topologically protected doublets can be realized by Josephson junction arrays [13, 14]. Although promising, this approach is difficult due to the required high degree of similarity of a large number of nanoscale junctions. Very recently it was shown that it is possible to realize these Hamiltonians by polar molecules in optical lattices, [16] but this is also technologically difficult. The long range Hamiltonian (1) has the advantage, besides the stronger protection described above, that it can be realized in a trapped ion system with present day technology using laser induced coupling proposed in [18] (see Fig. 1) . This type of interaction has already been used to create multiparticle entangled states in [21].

In this approach the physical system consists of $N^2$ ions that can either be arranged in a two dimensional array of microtraps, see Fig. 1, or in a linear trap, see Fig. 2. In both situations, strings of ions are illuminated by laser fields tuned close to a particular resonance frequency of individual ions[18, 19, 20]. In these conditions the relevant degrees of freedom of each ion can be represented by a pseudospin $S = 1/2$ variable. As discussed below one can choose the laser fields so to generate an effective Hamiltonian of these pseudospins coupling ions two-by-two. Such a pseudospin can be implemented using an electronic ground state and an excited level directly coupled by a single photon transition [22], in this case spontaneous emission will not bring the system out of the pseudo spin space. Furthermore, for long lived states of alkaline earth ions[28], the corresponding weak metastability induces a small noise term of the form (2),

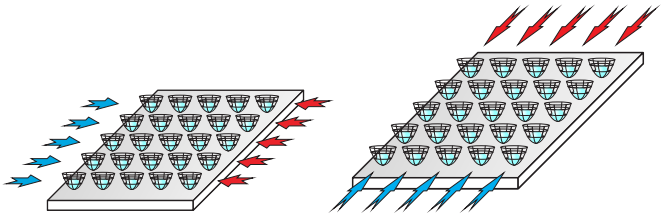

FIG. 1: Two laser beams of slightly different frequencies (blue and red) produce virtual processes that change the state of one ion and create (annihilate) a phonon mode of $N \times N$ lattice. Each of these laser fields alone does not produce a real ion transition (with or without a phonon mode emission or absorption). The sum of their frequencies is exactly equal to the energy needed to change the state of two ions simultaneously which is described by the $\sigma^{x,y}_{i,j}\sigma^{x,y}_{i,k}$ in the effective Hamiltonian. If the phonon mode involved in this process corresponds to global translational motion of all ions (center-of-mass mode), the effective pair-wise interaction does not depend on the distance between two ions. It can be restricted to the desired row (column) form if only a subset of ions is illuminated at a given time. The effect of the sequential application of the beams to rows and columns is described by the effective Hamiltonian 1.

which is efficiently treated by the incipient protection mechanism of Hamiltonian (1).

We now sketch the derivation of the effective Hamiltonian (1 ) and estimate realistic constraints on its parameters. Each ion is in one of two quantum states with an energy difference $\omega_{eg}$, the interaction between ions produce a collective vibrational mode of frequency $\nu$ corresponding to the global displacement of all ions. The ions are subjected to two laser fields of frequencies $\omega_{1,2} = \omega_{eg} \pm (\nu + \delta)$ and Rabi frequency $\Omega$. In the interacting representation the physical Hamiltonian describing the ion system in the $\omega = \omega_{eg} + (\nu + \delta)$ laser field is [19, 20] :

$$H_{int} = \Omega J_+ e^{-i(\nu+\delta)t} + \frac{i\eta\Omega}{\sqrt{2}}(a^\dagger + ae^{-2i\nu t})J_+ e^{-i\delta t} + h.c. \tag{4}$$

where $\eta$ is the Lamb-Dicke parameter defined as $(\hbar^2 k_j^2/2M\hbar\nu)^{1/2}$ and $a$ and $a^\dagger$ are the annihilation and creation operators of the phonon mode. Provided that $\Omega \ll \nu$ and $\eta \ll 1$ the main contribution to the effective Hamiltonian comes from the terms oscillating with frequency $\delta$. For small interaction constant $\eta\Omega \ll \delta$ these terms can be treated in the perturbation theory resulting in the desired $\chi(\mathbf{J}\cdot\mathbf{n})^2$ term of the effective interaction where $\mathbf{n}$ is a unit vector whose direction in $xy$ plane is controlled by the relative phases of the laser fields[18]. Generally, a stronger interaction might lead to the excitation of the phonon mode. This process can be suppressed[19] by choosing the time of the interaction $\tau$ in such a way that the phonon system returns to its initial state, $\tau\delta = 2\pi K$. In this case the effective interaction remains $\chi(\mathbf{J}\cdot\mathbf{n})^2$ with $\chi = \frac{\eta^2\Omega^2}{\delta}$.

Because the interaction occurs only between the ions illuminated at one time, the spatial form of the physical array does not need to be directly related to the column/row form of the effective Hamiltonian (1). Thus, as we illustrate in Fig. 2, a possible configuration rendering the implementation of the Hamiltonian easier in existing systems, consists of using a linear array of ions.

| | $2\times 2$ | $3\times 3$ | $4\times 4$ | $5\times 5$ |
|---|---|---|---|---|
| SRI | 0.84 $J_x$ | 0.58 $J_x$ | 0.32 $J_x$ | 0.20 $J_x$ |
| LRI | 0.84 $J_x$ | 0.96 $J_x$ | 0.92 $J_x$ | 0.80 $J_x$ |

TABLE I: Gap as a function of the array size for short range interaction (SRI) and long range interaction (LRI)

This model also assumes that the lasers interact only with mode of frequency $\nu$. This becomes difficult to achieve in large systems where the number of modes is large. Small coupling to these modes will induce a position dependent coupling in the effective Hamiltonian modifying only slightly the gap in the spectrum. Large coupling to these modes would lead to their excitation which is much more dangerous. To avoid this process we need to ensure that $\max(\delta, \Omega) < \nu - \nu_1$ where $\nu_1$ is the frequency of the closest mode. In the case of a string of ions, $\nu - \nu_1 \approx (1 - \sqrt{3})\nu$ and is roughly independent of the number of ions [23], for a fixed trapping potential. Thus, in order to maximize interaction, we need to adjust the parameters so that $\nu - \nu_1$ is maximal while keeping the ions at distances sufficiently far apart so that they can be addressed individually. This leads to conditions on the translational mode frequency, which can be satisfied experimentally even for chains of 8 ions [24]. Using the results of [23, 25] for the phonon spectrum of a single ion chain and assuming a fixed inter-ion distance, we find that $\nu - \nu_1 \approx \frac{1}{N^2} \times 10$ MHz, which implies that such implementation is feasible for $N \leq 3$.

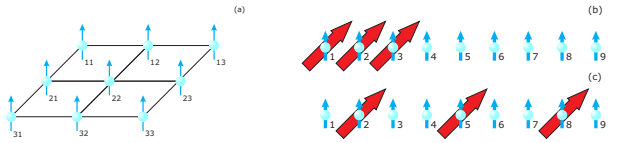


FIG. 2: An array of $3 \times 3$ implemented by a linear trap with 9 ions. To generate the interaction in row 1 (insert a) one applies laser light to ions 1, 2, 3 (insert b) while column 2 interaction requires light on ions 2, 5 and 8. (insert c). The advantage of this scheme is that it does not require micro trap fabrication. However, it becomes difficult to implement for large arrays ($N > 3$) because the distance between the ions in the center of the array gets smaller making their individual addressing difficult.

To implement larger arrays one can use a two dimensional configuration of surface traps of a few meV depth, similar to those used in [26]. When the traps are at a distance $a_{min}$ from each other the Coulomb repulsion becomes of the same order of magnitude as the trapping potential. In this case, one can achieve a frequency of the translational modes of the order of $\nu = 10$ MHz [26] and our numerical results show that for a $5 \times 5$ square array configurations $\nu - \nu_1 \approx 0.1\nu$. Coupling between rows and columns is then avoided by applying lasers that alternate between lines and columns. The resulting Hamiltonian is the sum of the terms in (1) provided that the duration of each pulse satisfies $\tau J_{x,y} \lesssim 1$. We summarize the experimental values and resulting couplings in Tab. II.

| | 4 ions | 9 ions | $5\times 5$ ions |
|---|---|---|---|
| $\Omega(Hz)$ | $10^5$ | $10^5$ | $10^6$ |
| $\delta(Hz)$ | $10^4$ | $10^4$ | $10^5$ |
| $J_{x,y}(Hz)$ | $10^4$ | $10^4$ | $10^5$ |

TABLE II: Parameters and induced coupling for the various arrays, $\nu = 1$ MHz for 4 and 9 ions and 10 MHz pour $5 \times 5$ ions, $\eta = 0.1$ in both cases, $J_{x,y} = \eta^2\Omega^2/\delta$, $K = 1$.

The strength of the induced interaction, $J_x, J_y$ and thus the gap in the spectrum $\Delta$ of the Hamiltonian (1) is limited by the condition that the laser do not lead to the excitation of spurious phonon modes. Thus, to

achieve a maximal interaction strength one needs a vibrational spectrum with the largest possible gap between the global translational mode and the rest of the spectrum. Another constraint comes from the condition that the distance from the ions is sufficiently large to allow for their individual addressing by laser beams. This gap is very sensitive to the geometry of the lattice. We have identified the two most promising candidates mentioned above: small (4 and 9 ions) one dimensional (Fig. 2) and larger ($5 \times 5$) highly symmetric two-dimensional structures (Fig. 1).

We now estimate the effective decoherence rates which can be obtained in these three systems in realistic conditions. Assuming that the allowed minimal separation between ions is $2\mu$m we get (Tab. II) that the induced couplings $J_{x,y}$ of the effective Hamiltonian are $10^4$ Hz for the linear array and $10^5$ Hz for the square array. As shown in Tab. I, the gap is of the same order of magnitude as the coupling coefficient in the isotropic case, $J_x = J_y$.

The effective decoherence rate (3) is determined by the largest noise term and by the single ion decoherence rate. In an ideal system where the only origin for noise would be the individual ion decoherence ($b_{i,j}^{x,y,z} \approx \Gamma_0$), the effective lifetime would be astronomical (assuming a minimal separation between ions $a_{min} = 2\mu m$ we get $10^{25}$s for a $5\times$ 5 array). In a more realistic situation, one must take into account noises induced by the ions manipulation with laser light: the dominant noise turns out to come from laser frequencies noise $\delta f$. Frequencies drift with time resulting in a $H = \hbar\delta f \sum_{i,j} \sigma_{i,j}^z$ term in the effective Hamiltonian,. Though one can suppress this noise down to the level of some Hz [27] we shall use a more realistic value of 500 Hz for our estimates summarized in Tab. III.

| | 4 ions | 9 ions | $5 \times 5$ ions |
|---|---|---|---|
| $\Gamma_{eff}(Hz)$ | $1.5 \cdot 10^{-3}$ | $7.5 \cdot 10^{-5}$ | $1.9 \cdot 10^{-11}$ |
| $\tau(s)$ | $6.6 \cdot 10^{2}$ | $1.3 \cdot 10^{4}$ | $5.3 \cdot 10^{10}$ |

TABLE III: Effective decoherence rates and topological qubit lifetimes.

Global state initialization can be achieved by first imposing a large effective field along the $x$ direction, yielding a large additional term proportional to $H_{\rm ext} = \pm \sum \hat{\sigma}_{i,j}^x$ in (1). This leads to the global state of the array $|0\rangle = \prod |0\rangle_{ij}$ or $|1\rangle = \prod |1\rangle_{ij}$ depending on the overall sign in $H_{\rm ext}$. These states satisfy $Q_j |0\rangle = |0\rangle$ and $Q_j |1\rangle = -|1\rangle$ (if $N$ is odd). When $H_{ext}$ is switched off adiabatically (i.e on a time scale much longer than $1/\Delta$), the final Hamiltonian becomes (1). At all stages of this evolution the Hamiltonian commutes with $Q_j$ operators. Thus, the two initial unprotected states $|0\rangle$ and $|1\rangle$ evolve into the protected states with the same quantum numbers. Note that $P_i$ operators commute only with the final Hamiltonian, and the initial states are not eigenstates for them. The final state can be reconstructed using standard methods [24] measuring the state of individual ions in the $\sigma^x$ or $\sigma^y$ basis, after the protecting interaction (1) has been switched off.

We have shown that it is possible to use the flexibility of ion traps to form long living quantum states topologically protected from decoherence, even small 4 ion strings being expected to give relaxation times one order of magnitude longer than currently achieved (Tab. III). It is quite likely that similar techniques can be used for experimental implementation of more exotic quantum states in controlled atomic systems, for instance the states described by the anyon statistics that would make possible to implement a full quantum computation in the protected subspace.

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